# **Does It Pay To Anticipate Competitor Reactions?**

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#### Abstract

Analyzing and anticipating competitor moves has been central to modern competitive strategy. In contexts involving intense interfirm interaction, the value of a particular strategy depends in large part on how competitors will react to it. Despite many developments, anecdotal evidence indicates that the effective use of techniques to gauge decisions based on competitive considerations has been scant in practice. Our paper intends to fill this void. Using data from the auto insurance industry in Brazil, we compare strategies that do and do not anticipate competitor reactions. Basically we show that it does pay to anticipate those reactions. An optimal strategy will explore both demand elasticities and competitors' patterns of reaction. We show that such "strategic" policy is expected to outperform a "myopic" approach which ignores competitor reactions. We also develop a methodology to compute demand elasticities, reaction functions and numerically compute optimal reaction strategies.

### Introduction

Analyzing and anticipating competitor moves has been central to modern competitive strategy. In contexts involving intense interfirm interaction, the value of a particular strategy depends in large part on how competitors will react to it. Thus, Porter (1980, p. 72) has proposed the development of a "competitive intelligence system" aimed at "compiling, cataloging, digesting, and communicating" data about competition. A flurry of research attempting to systematize the process of collection and analysis of data from competitors has recently emerged (e.g., Kahaner, 1996; Prescott and Miller, 2002; Warren, 2002). In parallel, several authors have sought to apply game-theoretic concepts to the assessment and prediction of competitor moves (e.g., Brandenburger & Nalebu., 1996; Ghemawat, 1991; Tirole, 1988). Despite those developments, anecdotal evidence indicates that the effective use of techniques to gauge decisions based on competitive considerations has been scant in practice. Bossidy and Charan (2002, p. 192-193), for instance, report the following case:

"...one December I had a call from a CEO of a \$5 billion company... One key division was responsible for the company's failure to meet its earnings forecast. The person who was running it [developed a strategy] to gain market share by cutting prices. . . He calculated that his increased volume from cutting prices would lower costs. When the CEO reviewed it, the strategy made sense to him. We went over all this and finally I asked, 'So what did you miss?' By then the CEO had figured it out. 'I did not ask him what the competitors' reactions would be,' he said. The biggest competitor matched the price cuts almost immediately, and the other followed. Prices for the entire industry went down. . . The CEO replaced the division head, and the new man he brought in gradually rolled the prices back up [and] competitors followed the price increases. . . "

This example illustrates that failing to anticipate competitive reactions can severely undermine the value of certain strategies. Thus, firms may be able to improve performance by developing practices to monitor the competition and act accordingly. Several factors, however, may plague firms' ability to effectively benefit from those practices. First, patterns of competitor reaction may be difficult to predict or may change over time. Second, the development and maintenance of competitive intelligence systems may be costly. At the most fundamental

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level, we need to ask whether firms can improve their performance by developing a systematic program to act upon estimates of competitive reactions, and assess the magnitude of such an improvement. Surprisingly, to our best knowledge, this simple question remains unexplored in the literature on competitive strategy.

Our paper intends to fill this void. Specifically, our research question is: Does an anticipation of future competitor reaction in current pricing decisions matter? Does it improve firm performance compared to a simpler approach without such an anticipation?" Using data from the auto insurance industry in Brazil, we assess whether a "reaction" pricing policy, which anticipates competitor reactions, outperforms a simpler "myopic" policy ignoring such reactions. To do so, in the next section we describe the two pricing policies that can be followed by our firm of interest: "myopic" and "strategic". In section 3 we present estimates of demand and competitor reactions curves. Those estimates are then used to compute and compare the two policies in section 4. The last section presents our final conclusions.

## **Pricing Policies**

In this section, we develop a pricing algorithm assuming a partial equilibrium framework for our firm of interest, denoted Firm 1. The idea is to compare two pricing policies: the "myopic" policy, with no anticipation of competitor reactions, and the "strategic" policy, with considerations on likely competitor reactions. Our pricing policies assume a partial equilibrium framework because we infer competitors' reactions from market data rather than use a specific objective function maximization, for example, maximization of profits or market share.

In the "myopic" policy, a specific firm assumes that no firm can individually affect prices of other competitors or industry prices. It is a common assumption of perfect competition markets. It is considered a "myopic" policy because Firm 1 does not look at other firms' reactions to determine its price changes. This strategy is easy to implement and needs limited information. Basically, Firm 1 only needs the estimation of its own demand curve. Finally, taking competitors' prices as given would be a very reasonable assumption if the industry were in perfect competition. That is, if the industry is composed of many small firms that cannot affect market prices individually.

The potential drawback of the "myopic" strategy is that Firm 1 and competitors' size can matter. If the competition is aggressive and market is concentrated on few players, one firm may observe and react to price changes of other firms. One firm may try to expand its market share by reducing prices, but if the competitors follow suit, the price change can initiate a price war, and consequent decrease in market prices. The final result of this price reduction may be smaller profits and the same initial market share. The example given in the introduction illustrates this point very well.

Therefore, we also define the "strategic" policy, which assumes that Firm 1 does not take other firms' prices as given. In this case, Firm 1 will choose an optimal pricing policy taking into account that its price decisions will affect other firms' prices, which in turn will also affect its own demand. The advantage of this method is that it is more general and does not rely on the assumption of perfect competition. However, in order to compute this strategy, we will need a large amount of data about other firms' price history.

### "Myopic" Policy

We assume that the main objective of Firm 1 is to maximize the present value of its profits. To simplify our problem, we limit the time horizon to a finite number of T periods. Prices of the other competitors are assumed to be constant from one period to the other. Therefore, Firm 1 "myopic" policy will solve the following problem:

$$Max_{\{p_{t}^{1}\}}\sum_{t=0}^{T} \delta^{t} Q(P_{t}^{1}, P_{t}^{2}, \dots, P_{t}^{N})(P_{t}^{1} - S_{t}^{1})...$$
(P1)  
s.t.:  $P_{t}^{j} = P_{t-1}^{j}$   $j = 2, \dots, N; t = 1, \dots, T,$ 

Where  $\delta$  is the discount rate, Q (.) is the demand function, which depends on the price that Firm1 has at time t,  $P_t^j$ , and N – competitors' prices  $P_t^2, \ldots, P_t^N$  at time t, and  $S_t^1$  represents the unitary cost for Firm 1.

One can argue that the theoretical model described in (P1) is unrealistic for three main reasons. First, Firm 1 does not observe directly its demand function. Second, market is dynamic and the demand curve will not be a stable function across time. Third, companies choose pricing policies recursively; at each period they choose only the price for the next period instead of determining the whole sequence of future prices.

In order to overcome this criticism, we suggest a modified version of problem (P1). We assume that Firm 1 will estimate its demand function using all available market information. Thus, Q will be replaced with an estimated function from available data, denoted by  $\hat{Q}$ . Second, since Firm 1 will use estimated values for Q, in each period as new data arrive, those estimates can be improved. This means that the demand curve will be a function of time and will be updated each period; we use then the notation  $\hat{Q}_t$  to represent this time-dependent estimated demand function. Finally, we will model the optimal strategy used by Firm 1 in a recursive way: in each period, will choose its next period price and postpone the remaining sequence of prices until the future. In the next period, Firm 1 updates information about demand estimates and competitors' prices in order to choose next period prices. Those assumptions are summarized in the algorithm below:

#### "Myopic" algorithm

Step1: At each time s < T, s = 1, 2, ..., T, Firm 1 estimates the demand curve parameters using available data up to s - 1, thus obtaining the estimated function:

$$\hat{Q}_s = \hat{Q}_s \left( P_s^1, P_s^2, \dots, P_s^N; \Psi_s \right)$$
<sup>(1)</sup>

Where  $\Psi_s$  represents the additional sets of control variables used in (1).

In the "myopic" case, Firm 1 will assume no price changes from its competitors<sup>1</sup>, that is:

$$P_s^j = P_{s-1}^j \quad j = 2, \dots, N$$
 (2)

Step 2: Firm 1 will choose optimal pricing for period  $P_s^1$  where  $P_s^1$  is the first element of the price sequence  $\{P_{s+j}^1\}_{j=0}^T$  that solves:

$$Max_{\{P_{t}^{1}\}}\sum_{t=0}^{T} \delta^{s+t} \hat{Q}_{s+t} \left(P_{s+t}^{1}, P_{s+t}^{2}, \dots, P_{s+t}^{N}; \Psi_{s+t}\right) \left(P_{s+t}^{1} - S_{s+t}^{1}\right) \dots$$
(MP)  
s.t:  $P_{s+t}^{j} = P_{s+t-1}^{j} \quad j = 2, \dots N; \ t = 0, 1, \dots, T$ .  
given:  $\left(P_{s-1}^{1}, P_{s-1}^{2}, \dots, P_{s-1}^{N}\right)$ .

Step 3: If time  $s + 1 \le T$  return to step 1, otherwise stop.

### Strategi Policy

We start from a similar problem as described in (P1); however, we no longer assume competitors' price as given. Instead, we assume that each firm k will change prices in response to past price changes of all firms, including its own change. In this setting, the initial problem can be stated as:

$$Max_{\{P_{t}^{1}\}}\sum_{t=0}^{T} \delta^{t} Q(P_{t}^{1}, P_{t}^{2}, ..., P_{t}^{N})(P_{t}^{1} - S_{t}^{1})$$
(P2)  
s.t.:  $\Delta P_{t}^{k} = R^{k} (\Delta P_{t-1}^{1}, \Delta P_{t-1}^{2}, ..., \Delta P_{t-1}^{N}) \quad k = 2, ..., N \quad t = 0, 1, ..., T$   
given:  $(P_{t-1}^{1}, P_{t-1}^{2}, ..., P_{t-1}^{N})$ 

Where  $\Delta$  is the delta operator that takes the first difference,  $\Delta P_t^j = P_t^j - P_{t-1}^j$ .

The reaction functions  $\Delta P_t^k = R^k \left( \Delta P_{t-1}^1, \Delta P_{t-1}^2, \dots, \Delta P_{t-1}^N \right)$ ,  $k = 2, \dots, N$ , denote competitors' pricing policy in response to past price changes of the entire market. We impose no restrictions on the coefficients of the reaction functions, we simply attempt to recover competitors' pattern of reaction from market data. We use first differences in order to avoid problems with unit roots.

It is immediate to have the same criticism made with regard to P1 and P2. Therefore, we will assume that Firm 1 uses estimated functions from available data for Q and R and will update them periodically. The algorithm also implies a recursive strategy, which is described below.

"Strategic" algorithm

Step1: At each time s < T, s = 1, 2, ..., T, Firm 1 estimates the demand curve parameters using available data up to s - 1, thus obtaining the estimated function:

$$\hat{Q}_{s+j}(P_{s}^{1}, P_{s}^{2}, \dots, P_{s}^{N}; \Psi_{s})$$
(3)

$$\Delta P_{t}^{k} = \hat{R}^{k} \left( \Delta P_{s-1}^{1}, \Delta P_{s-1}^{2}, \dots, \Delta P_{s-1}^{N}; \Phi_{s} \right) \quad k = 2, \dots N$$
<sup>(4)</sup>

where  $\Psi_s$ ,  $\Phi_s$  are additional sets of control variables used in the estimation realized in (3) and (4).

Step 2: Firm 1 will choose optimal pricing for period s,  $P_s^1$ , where  $P_s^1$  is the first element of the price sequence  $\{P_{s+k}^1\}_{k=0}^T$  that solves:

$$Max_{\{P_{t}^{1}\}}\sum_{t=0}^{T} \delta^{s+j} \hat{Q}_{s+j} (P_{s+j}^{1}, P_{s+j}^{2}, \dots, P_{s+j}^{N}; \Psi_{s+j}) (P_{s+j}^{1} - S_{s+j}^{1}) \dots$$

$$s.t. : \Delta P_{s+j}^{k} = \hat{R}_{s+j}^{k} (\Delta P_{s+j-1}^{1}, \Delta P_{s+j-1}^{2}, \dots, \Delta P_{s+j-1}^{N}; \Phi_{s+j}) \quad k = 2, \dots N$$
(RP)

Step 3: If time  $s + 1 \le T$  return to step 1, otherwise stop.

Notice that, for both strategies, step 2 indicates that, at each time t, Firm 1 will pick just the first price of the optimal pricing sequence  $\{P_{s+j}^1\}_{i=0}^T$ . The reason for this is that Firm 1 will always update information about the demand and "strategic" curves and use it in the problems described in MP and RP.

### Empirical Estimates: A Case Study on the Insurance Industry

In order to apply our model, we collected data from the auto insurance industry in Brazil. Basically, we estimated demand and reaction functions from a set of firms in this industry and then applied the "myopic" and "strategic" policies to a given firm.

Before we present our methodology, the reader should be acquainted with some information about this industry. In Brazil, the auto insurance industry observes a high level of competition among firms. Insurance companies usually work at very low margins and most of their profit comes from the financial margins of the collected premiums, prices paid by customers for the policies.

Prices change very dynamically; each month, firms review their pricing policies and define a new set of prices for each product. Notice that in the insurance business each different product is defined by the combination of the car model, year of production and location of use. The reason for this is that those three variables explain most differences in expected loss value, namely, the expected loss times the probability of loss.

Because it is very competitive and prices are very volatile over time, the auto insurance industry presents itself as a natural candidate to gauge our strategic model. To do that, our first hurdle was to select a benchmark firm, to estimate its demand function and also estimate reaction functions for the whole industry. This is what we perform in this section. In the following subsections, we discuss the data, estimation methodology and results for demand and reaction functions.

#### Data

The two cases studied reflect the same car model and year but involve two different locations in Brazil. Our selection was based on data availability and high volume markets, capturing a more dynamic competitive environment. Together with the pricing policy of Firm 1, we selected four major competitors, which correspond to more than 80% of the market.

Firm 1 has information only about its own demand and prices and observes only competitors prices, instead of competitors' demand. Our data have a panel structure (e.g. Wooldridge, 2002), in which Firm 1 demand and the insurance prices and competitors' insurance prices were monthly recorded for almost three years.

The data are organized in delimited groups defined by region and car models that have similar characteristics regarding brand, engine power, geographic region and market price. Each element within a group, therefore, involves a specific model, in a particular region, at a given time. We will call each group an auto-group, which we denominated AG1 and AG2. The models were adjusted independently for each auto group.

### **Demand Curves**

Total demand was split into two parcels. The first one, which we refer to as the "external demand", is related to policies sold to new customers; the other, which we name the "internal demand", considers only policies that were renewed by current customers. Each parcel was modeled separately within an auto-group. Information regarding market prices of the cars and the loss ratio<sup>2</sup> was used as instrumental variables whenever necessary.

We overcame the problem of identification by assuming that the firms will attend any demand to a given determined price, which is appropriate in our industry context. For the sake of simplicity, we will focus our study only on the "external demand" estimates, and we will refer to it simply as demand<sup>3</sup>.

For the demand function, two models were constructed. The first model is linear and is given by:

$$Q_{it}^{E} = \alpha + \beta^{1} P_{it}^{1} + \dots + \beta^{5} P_{it}^{5} + G_{i} + M_{t} + \varepsilon_{it}$$
(D1)

in which,  $Q_{it}^E$  is the demand of the first company at time t for element i of a given auto group,  $P_{it}^j$  is the price (deflated to the first month level) of the insurance policy for the element i at time t,  $G_i$  is the fixed effect of element i,  $M_t$  is the effect of the month of the year, and  $\mathcal{E}_{it}$  is the random error and  $\alpha, \beta^1, \dots, \beta^5$  are other parameters of the model.

Despite the simplicity of the model described in D1, it resulted in a poor in-sample fit. One of the possible reasons is that demand data had small variability and assumed values close to zero or even zero in some months. This fact led to negative predictions of the demand, which suggested the inadequacy of the linear model. To avoid this problem, we adopted a second specification which uses an exponential quasi-likelihood model with variance function similar to a negative binomial distribution (e.g., Wooldridge, 2002). The new model is:

$$E(Q_{it}^{E}) = \exp\{\alpha + \beta^{1} P_{it}^{1} + \dots + \beta^{5} P_{it}^{5} + G_{i} + M_{t}\}$$
 (D2)

with elements as defined in (D1).

Table 1 shows demand estimates of model D2 for two auto-groups, defined as AG1 and AG2 for Firm 1. Since our price decision algorithms assume that demand estimates are updated each period Firm 1 has to take a price decision, we present the whole set of estimates with data up to one period behind. In each estimate, we have observed negative elasticities to Firm 1's own price as expected. However, in both cases, cross-elasticities present negative and positive signs, although only non-negative coefficients should be expected, given that policies from different competitors are assumed to be substitute products.



#### Table 1 - demand estimates for ag 01 and ag 02

### **AG1 - Demand Curves**

Sample from: to:	Aug-02 May-05	Aug-02 Apr-05	Aug-02 Mar-05	Aug-02 Feb-05	Aug-02 Jan-05	Aug-02 Dec-04	Aug-02 Nov-04
	s=1	s=2	s=3	s=4	s=5	s=6	s=7=T
Constant	4.3106***	4.3689***	4.5471***	5.2134***	5.6401***	5.7435***	5.7007***
Firm_1	-0.0024***	-0.0022***	-0.0017 *	-0.0014 *	-0.0016 *	-0.0014 *	-0.0012 *
Firm_2	0.0005 *	0.0003 *	-0.0003 *	-0.0012 *	-0.0022 *	-0.0025 *	-0.0023 *
Firm_3	0.0008 *	0.0009 *	0.0010 *	0.0009 *	0.0008 *	0.0007 *	0.0005 *
Firm_4	-0.0032***	-0.0035***	-0.0042***	-0.0052***	-0.0051***	-0.0052***	-0.0052***
Firm_5	-0.0005 *	-0.0003 *	0.0002 *	0.0012 *	0.0019 *	0.0022 *	0.0019 **
R <sup>2</sup>	0.0823	0.0773	0.0712	0.0624	0.0688	0.0689	0.0749

## AG2 - Demand Curves

Sample from: to:	Aug-02 May-05	Aug-02 Apr-05	Aug-02 Mar-05	Aug-02 Feb-05	Aug-02 Jan-05	Aug-02 Dec-04	Aug-02 Nov-04
	s=1	s=2	s=3	s=4	s=5	s=6	s=7=T
Constant	1.9686***	2.0651***	2.4281***	3.3427***	3.8355***	4.4833***	4.7627***
Firm_1	-0.0033***	-0.0032***	-0.0034***	-0.0037***	-0.0043***	-0.0049***	-0.0051***
Firm_2	0.0011 *	0.0011 *	0.0006 *	-0.0005 *	0.0001 *	-0.0003 *	-0.0006 *
Firm_3	0.0006 *	0.0005 *	0.0003 *	-0.0004 *	-0.0003 *	-0.0005 *	-0.0006 *
Firm_4	0.0007 *	0.0006 *	0.0004 *	0.0000 *	-0.0003 *	-0.0003 *	-0.0003 *
Firm_5	-0.0013 *	-0.0012 *	-0.0004 *	0.0015 *	0.0012 *	0.0019 *	0.0023 *
R <sup>2</sup>	0.0310	0.0303	0.0339	0.0440	0.0460	0.0485	0.0527

Obs.: \*\*\*, \*\*, \* indicates that the parameter is statistically significant respectively to 99%, 95% and 90% confidence level

#### **Reaction curves**

The aim of the reaction curves is to predict the price variation of a company based on the price variations of the other competitors. As we have five companies involved in the problem, we have to estimate a set of five models, one for each firm<sup>4</sup>, given by:

$$\Delta P_{it}^{j} = \beta_{0}^{j} + \beta_{1}^{j} \Delta \frac{P_{it-1}^{1} + \dots + \beta_{5}^{j} \Delta P_{it-1}^{5} + \beta_{6}^{j} r_{t} + G_{i} + M_{t} + \varepsilon_{it}^{j} \quad j = 1, \dots, 5$$
(R1)

with  $P_{it}^{j}$  representing the price of the insurance of the element i and competitor j at time t,  $\Delta P_{it}^{j} = P_{it}^{j} - P_{it-1}^{j}$ , r<sub>t</sub> is the basic

interest rate,  $\mathcal{E}_{it}^{j}$  is a random error,  $\beta_{a}^{j}, \ldots, \beta_{b}^{j}$  are parameters and G<sub>i</sub> and M<sub>t</sub> are as defined in the last section.

Due to technical limitations, each model was estimated independently from the others. We used the estimation procedure proposed by Arellano (2003) for dynamic panel data. Tables 2 and 3 present reaction curves estimates for AG1 and AG2.<sup>5</sup> Like we did for the demand functions, we present seven different estimations that increase the sample size recursively as specified in the algorithms from section 2.

Tables 2 and 3 show the complexity of relationships among many firms reacting to its own price changes and four other firms' price changes. In order to get some intuition behind those results, we define some reaction strategies that one firm can pursue. First, let us define strategies that one firm can follow with respect to its own price:

Definition 2 (Inertial Strategy) A firm is considered inertial with respect to its own price if current increases (decreases) in price are preceded by past increases (decreases) in its own price. In other words, if its reaction coefficient to its own past price changes is significantly positive.

Definition 3 (Cyclical Strategy) A firm is considered cyclical with respect to its own price if current increases (decreases) in price are preceded by past decreases (increases) in its own price. In other words, if its reaction coefficient to its own past price changes is significantly negative.

Those two definitions present two different strategic patterns. In the case of the inertial firm, increasing (decreasing) price in the past means that the firm will continue to increase (decrease) prices in the present. A firm is said to have a cyclical strategy if it oscillates back and forth by increasing and decreasing prices.

We also define some reaction strategies of one firm with respect to other firms: it can follow, anti-follow or ignore the other firm.

Definition 4 (Follow Strategy) A firm A is said to follow firm B if it increases (decreases) its price after firm B price increases (decreases) its price. In other words, if its reaction coefficient to its own past price changes is negative. Or if its reaction coefficient to firm B past price changes is significantly positive.

Definition 5 (Anti-follow Strategy) A firm A is said to anti-follow firm B if it increases (decreases) its price after firm B price decreases (increases) its price. In other words, if its reaction coefficient to firm B past price changes is significantly negative.

Definition 6 (Ignore Strategy) A firm A is said to ignore firm B if it neither follows nor competes with firm B.

Analyzing the coefficient estimates from table 2 and 3, we can classify firms into the above defined strategies<sup>6</sup>. Let us consider only repeated strategies in both auto groups studied, AG1 and AG2. Firm 1 is inertial regarding its own price and anti-follows firm 5. Firm 2 is inertial regarding its own price, and follows Firm 1 and anti-follows 4. Firm 3 is also inertial regarding its own price, but presents no consistent strategy with respect to its competitors. In the case of Firm 4 and Firm 5, both do not show a specific strategy with respect to its own price, but the former follows Firm 1. The results indicate that in general each firm presents an inertial strategy with respect to its own price. Furthermore, firms in general choose one benchmark firm and decide on the strategy of following or anti-following this benchmark firm. One possible interpretation is that one firm follows another firm trying to replicate the same price policy strategy and keeping its relative market share constant to the benchmark. In contrast, an anti-follow strategy means that one firm goes in the opposite direction, decreasing prices when the benchmark firm increases its price<sup>7</sup>, possibly intending to get a market share from the other firm.

	Sample from:	Aug-02	Aug-02	Aug-02	Aug-02	Aug-02	Aug-02	Aug-02
	to:	May-05	Apr-05	Mar-05	Feb-05	Jan-05	Dec-04	Nov-04
Depend. Variable	Indep. Variable	s=1	s=2	s=3	s=4	s=5	s=6	s=7=T
	Firm_1	0.868***	0.902***	1.013***	0.751***	0.819***	0.770***	0.627***
	Firm_2	0.264 **	0.033	0.054	0.191 **	-0.031	-0.013	0.074
Firm 1	Firm_3	-0.131	-0.077	-0.081	0.076	0.250	0.191	0.004
	Firm_4	-0.153 *	-0.111 *	-0.104	-0.060	0.098	0.080	-0.093
	Firm_5	-0.2 <mark>69***</mark>	-0.1 <mark>53***</mark>	-0.15 <mark>4**</mark> *	-0.157***	-0.097***	<mark>-0</mark> .086***	-0.080 **
	Firm_1	0.570***	0.558***	0.55 <mark>2***</mark>	0.435 **	0.397 **	0.369***	0.279
	Firm_2	0.351***	0.662***	0.619***	0.761***	0.745***	0.652***	0.813***
Firm_2	Firm_3	0.041	-0.049	-0.023	-0.012	0.069	0.014	-0.001
	Firm_4	-0.064	- <mark>0.1</mark> 08 **	-0.16 <mark>7 **</mark>	-0.197***	-0.189***	-0.184***	-0.108
	Firm_5	-0.061 **	-0.166***	-0.108 **	-0.096 **	-0.071 **	-0.033	-0.081 **
	Firm_1	0.460***	0.678***	0.338 **	0.204	0.296	0.332 **	-0.112
	Firm_2	-0.272 **	-0.393***	-0.294 **	-0.303 **	-0.473***	-0.554***	-0.139
Firm_3	Firm_3	0.301***	0.207***	0.344	0.339***	0.319***	0.269	0.336***
	Firm_4	0.275 **	0.288***	0.350***	0.347***	0.378***	0.453***	0.382
	Firm_5	-0.148***	<b>-0.101***</b>	-0.073***	-0.017	0.040	0.034	-0.080***
	Firm_1	0.669***	0.714***	0.674***	0.660***	0.738 **	0.602***	0.434
Firm_4	Firm_2	0.100	0.091	0.322	-0.148	0.217	0.013	0.341
	Firm_3	0.060	0.074	0.111	0.025	0.307	0.171	0.382
	Firm_4	0.202***	0.197 **	0.131	0.059	0.093	0.152	0.279
	Firm_5	-0.172***	-0.181***	-0.185	-0.044	-0.079	-0.020	-0.181 **
Firm_5	Firm_1	-0.158 **	-0.246***	-0.092	-0.073	-0.078	-0.092	-0.118
	Firm_2	0.266	0.653 **	0.952***	1.014***	0.915***	1.034***	0.883***
	Firm_3	-0.080	-0.079	-0.112	-0.117 **	-0.131***	-0.196	-0.219
	Firm_4	-0.207 **	-0.449***	-0.304 **	-0.289 **	-0.182 **	-0.206 **	-0.108
	Firm_5	-0.129***	-0.223 **	-0.359***	-0.380***	-0.331***	-0.419***	-0.385***

Table 2 - reaction estimates for ag1

Obs.: \*\*\*, \*\*, \* indicates that the parameter is statistically significant respectively to 99%, 95% and 90% confidence level

Finally, reaction strategies can be used to induce the market to follow one company's strategy, for example, in AG1, Firm 1 is followed by firms 2 and 4, and firms 3 and 5 follow Firm 2. Therefore, Firm 1 can make all firms follow its strategy by leading 2 and 4 directly and 3 and 5 indirectly through Firm 2.

Those analysis, although intuitive, are not enough to define a specific optimization strategy, since we have to consider reactions in direct and indirect order, and also look at the magnitude of those reactions and their specific demand elasticities. In order to do such complex task we use a numerical method which is presented in the next section.

## Does it pay to anticipate?

Our approach is to apply the algorithm described in section 2 using demand and reaction estimates of section 3. The numerical results were obtained by a piece of software with programming in Matlab 6.5. We use the mathematical model for demand with binomial negative fit of parameters described in equation D2 and reaction functions estimated by equation (R1). The solution for the optimization problems described in the "myopic" and "strategic" algorithms is obtained from Matlab software using finincon.m code. This code uses the Nelder-Mead algorithm<sup>8</sup> to search the optimal point for maximum or minimum of functions. The computer program was made in Matlab version 6.5.

Sample	Aug-02	Aug-02	Aug-02	Aug-02	Aug-02	Aug-02	Aug-02
from:							
to:	May-05	Apr-05	mar/05	Feb-05	jan/05	Dec-04	nov/04
Indep.	s=1	s=2	s=3	s=4	s=5	s=6	s=7=T
Variable							
Firm_1	0.666***	0.681***	0.373 **	0.247	0.594***	0.489***	0.187
Firm_2	-0.016	-0.084 *	0.348	0.037	-0.062	0.026	0.01
Firm_3	0.082 **	0.065***	0.299 *	0.199***	0.005	0.110 **	0.088
Firm_4	0.098 *	0.070 *	0.216 **	0.192	-0.038	0.018	-0.073 *
Firm_5	-0.634***	-0.564***	-1.095***	-0.698***	-0.186	-0.400***	-0.144
Firm_1	0.26	0.401 *	0.327***	0.056***	0.189***	0.085 *	-0.037 *
Firm_2	0.469 *	0.473 *	0.496 *	0.252 *	0.359 *	0.330 *	0.163 *
Firm_3	0.062***	0.114 *	0.097***	-0.017***	-0.031***	-0.028***	0.016 *
Firm_4	-0.045***	-0.051***	-0.080***	0.017***	-0.2 <mark>37*</mark> **	-0.181***	-0.015 *
Firm_5	0.161***	0.195***	0.2 <mark>66***</mark>	0.176***	0.490 *	0.418 *	0.250 **
Firm_1	-0.283***	0.017***	-0.155***	-0.145***	-0.255***	-0.692 *	-0.585 *
Firm_2	0.078***	-0.054***	0.045***	0.008***	0.159***	0.108***	0.043 *
Firm_3	0.545 *	0.440 *	0.508 *	0.486 *	0.577 *	0.644 *	0.565***
Firm_4	0.03 <mark>5***</mark>	0.276***	0.444 *	0.294***	0.394***	0.391 **	0.346***
Firm_5	-0.1 <mark>22***</mark>	0.114***	-0.253***	-0.059***	-0.474***	-0.449 *	-0.333 **
Firm_1	0.326 *	0.223 *	0.235 *	0.230 *	0.269 *	0.265 *	0.278 *
Firm_2	0.321 *	0.448 *	0.128***	0.264 **	0.159 **	0.192 *	-0.102***
Firm_3	-0.077***	0.029***	-0.130***	0.043***	-0.214 *	-0.194 *	-0.333 *
Firm_4	0.760 *	0.892 *	0.870 *	1.100 *	0.523 *	0.521 *	0.826 *
Firm_5	-0.715 *	-0.922 *	-0.442 *	-0.630***	0.059***	0.024***	0.122***
Firm_1	-0.059***	0.140***	0.143***	0.119***	0.181***	0.152 *	0.286 *
Firm_2	-0.027***	0.097***	0.095***	-0.013***	-0.054***	-0.069***	0.010***
Firm_3	0.053***	0.046***	0.047***	-0.024***	-0.056***	-0.025***	-0.007***
Firm_4	0.021***	-0.238***	-0.207***	-0.198***	-0.243 *	-0.010***	-0.164***
Firm_5	0.618 *	0.894 *	0.887 *	0.927 *	1.026 *	0.783 *	0.857 *
	from: to: Indep. Variable Firm_1 Firm_2 Firm_3 Firm_4 Firm_5 Firm_1 Firm_2 Firm_3 Firm_4 Firm_5 Firm_1 Firm_2 Firm_3 Firm_4 Firm_5 Firm_1 Firm_2 Firm_3 Firm_4 Firm_5 Firm_1 Firm_2 Firm_3 Firm_4 Firm_5 Firm_1 Firm_2 Firm_3 Firm_4 Firm_5 Firm_1 Firm_2 Firm_3 Firm_4 Firm_5 Firm_1 Firm_2 Firm_3 Firm_4 Firm_5 Firm_1 Firm_2 Firm_3 Firm_4 Firm_5 Firm_1 Firm_2 Firm_3 Firm_1 Firm_2 Firm_3 Firm_1 Firm_2 Firm_3 Firm_4 Firm_5 Firm_1 Firm_2 Firm_3 Firm_4 Firm_3 Firm_1 Firm_2 Firm_3 Firm_1 Firm_2 Firm_3 Firm_1 Firm_3 Firm_1 Firm_2 Firm_3 Firm_1 Firm_3 Firm_4 Firm_5 Firm_1 Firm_3 Firm_4 Firm_5 Firm_1 Firm_3 Firm_4 Firm_5 Firm_1 Firm_3 Firm_4 Firm_5 Firm_1 Firm_3 Firm_4 Firm_5 Firm_1 Firm_3 Firm_4 Firm_5 Firm_1 Firm_3 Firm_4 Firm_5 Firm_1 Firm_3 Firm_4 Firm_3 Firm_4 Firm_3 Firm_4 Firm_3 Firm_4 Firm_3 Firm_4 Firm_3 Firm_4 Firm_5 Firm_1 Firm_3 Firm_4 Firm_5 Firm_1 Firm_3 Firm_4 Firm_5 Firm_1 Firm_4 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_1 Firm_5 Firm_5 Firm_5 Firm_5 Firm_5 Firm_5 Firm_5 Firm_5 Firm_5 Firm_5	Sample       Aug-02         from:       May-05         Indep.       s=1         Variable       Sell         Firm_1       0.666***         Firm_2       -0.016         Firm_3       0.082 **         Firm_4       0.098 *         Firm_5       -0.634***         Firm_1       0.26         Firm_2       0.062***         Firm_3       0.062***         Firm_1       -0.283***         Firm_1       -0.283***         Firm_1       0.035**         Firm_3       0.545 *         Firm_4       0.035***         Firm_5       -0.122***         Firm_1       0.326 *         Firm_2       0.321 *         Firm_4       0.760 *         Firm_5       -0.715 *         Firm_1       -0.059***         Firm_2       -0.027***         Firm_3       0.053***         Firm_3       0.053***         Firm_3       0.053***         Firm_1       -0.021***         Firm_4       0.021***	Sample       Aug-02       Aug-02         from:       May-05       Apr-05         Indep.       s=1       s=2         Variable       0.666***       0.681***         Firm 1       0.666***       0.681***         Firm 2       -0.016       -0.084 *         Firm 3       0.082 **       0.065***         Firm 4       0.098 *       0.070 *         Firm 5       -0.634***       -0.564***         Firm 1       0.26       0.401 *         Firm 2       0.469 *       0.473 *         Firm 3       0.062***       0.114 *         Firm 4       -0.045***       -0.051***         Firm 5       0.161***       0.195***         Firm 1       -0.283***       0.017***         Firm 2       0.078***       -0.054***         Firm 3       0.545 *       0.440 *         Firm 4       0.035***       0.276***         Firm 5       -0.122***       0.114***         Firm 5       -0.122***       0.114***         Firm 4       0.326 *       0.223 *         Firm 3       -0.077***       0.029***         Firm 4       0.760 *       0.892 *	Sample from:Aug-02Aug-02Aug-02Aug-02from:May-05Apr-05mar/05Indep.s=1s=2s=3Variables=1s=2s=3Firm 10.666***0.681***0.373 **Firm 2-0.016-0.084 *0.348Firm 30.082 **0.065***0.299 *Firm 40.098 *0.070 *0.216 **Firm 5-0.634***-0.564***-1.095***Firm 10.260.401 *0.327***Firm 20.469 *0.473 *0.496 *Firm 30.062***0.114 *0.097***Firm 4-0.045***-0.051***-0.080***Firm 50.161***0.195***0.266***Firm 1-0.283***0.017***-0.155***Firm 20.078***-0.054***0.045***Firm 30.545 *0.440 *0.508 *Firm 40.035***0.276***0.444 *Firm 5-0.122***0.114***-0.253***Firm 5-0.122***0.114***-0.253***Firm 10.326 *0.223 *0.235 *Firm 3-0.077***0.029***-0.130***Firm 40.760 *0.892 *0.870 *Firm 5-0.715 *-0.922 *-0.442 *Firm 1-0.059***0.140***0.143***Firm 2-0.027***0.097***0.095***Firm 30.053***0.046***0.047***Firm 40.021*** <td< th=""><th>Sample from:Adg-02Adg-02Adg-02Adg-02Adg-02from:May-05Apr-05mar/05Feb-05Indep. 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Table 3 - reaction estimates for ag 02

Obs.: \*\*\*, \*\*, \* indicates that the parameter is statistically significant respectively to 99%, 95% and 90% confidence level

A sensible gain in expected profits by pursuing the "strategic" policy instead of the "myopic" policy is interpreted as the gain of anticipating competitor reactions. Our simulation results show that, at least for the two cases we studied, it does pay to anticipate competitor reactions. The expected gain in AG1 is an increase of 26%, while the expected gain in AG2 is an increase of 10% in profits. Those expected gains are to be considered relevant for an industry that works at very low margins and depends on financial gains on collected premiums to attain positive margins.

Figures 1 and 2 give a better intuition of the "strategic" policy. In the case of AG1 (figure 1), Firm 1 strategy is to increase prices by an average of 40% from period three to nine<sup>9</sup>. It makes all the other firms increase average prices too; thus, firms 2, 3, 4 and 5 increase prices on average by 19%, 5%, 18% and 12% respectively. We can draw two results from this: First, Firm 1's strategy to maximize profits is to increase prices to explore its demand elasticity. Second, this increase in prices can be higher when Firm 1 anticipates the competitors' reactions since they will follow this price increase, that is, there is a "reverse price war".

The opposite strategy is presented in AG2. In this case (figure 2), Firm 1 decreases prices from period three to nine on an average of 12%. This decrease in prices is not followed by a price war since the other firms change prices very slightly; thus, firms 2, 3, 4 and 5 change prices on average by respectively -4%, 2%, -3% and -3%. In this case, cutting prices gives a better profit because the strategy explores the demand elasticity and does not trigger a price war.

#### Conclusions

In this paper, we showed that considering competitor reactions can play a major role in maximizing profits. We studied two groups of insurance policies, called AG1 and AG2 (auto group 1 and 2), in the auto insurance industry in Brazil and estimated a demand curve for the reference firm and reaction functions for its major competitors.

One of the main results is to show that a "strategic" policy will explore both its estimated demand elasticities and the expected pricing policy followed by its competitors. When an optimal "strategic" policy involves price increases, it will also induce competitors to increase prices in some kind of "reverse price war". If the optimal "strategic" policy involves a decrease in prices, it can avoid a price war looking at expected competitors' pricing dynamics as dictated by their estimated reaction curves.

The main question is when and how to make the price changes. For this, it is crucial to compute demand elasticities, reaction functions coefficients and numerically compute strategies. Our main contribution is to describe a methodology to do that empirically and compositionally. In the end, the answer to the question posed in the title is yes, it does pay to anticipate competitors moves and to use the information strategically.







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## Endnotes

<sup>1</sup> A more general hypothesis is to assume that Firm 1 will expect competitors prices to follow a random walk, that is  $P_s^j = E_s \left( P_{s-1}^j \right)$ , where  $E_s \left( \right)$  denotes expected value conditioned on information available at time s. Although this approach is more general, it would also imply a stochastic problem instead of a deterministic one. Assuming that our hypothesis in equation (2) has the same intuition of expecting no price changes, for the sake of simplicity, we will use the formulation described in (2).

<sup>2</sup> In the insurance industry, the loss ratio is calculated as value paid as indemnities divided by the total revenue.

<sup>3</sup> Previous analyses indicated that the "internal demand" is very inelastic, thus, ignoring it from our subsequent computation should not be problematic.

<sup>4</sup> For Firm 1, estimation of equation R1 was just to investigate its reaction pattern against its competitors. This estimation is not necessary for the application of the algorithms described in section 2.

<sup>5</sup> In those tables, we also include Firm 1, although the algorithm will assume that Firm 1

will no longer use that strategy.

<sup>6</sup> We used an arbitrary criterion of considering a firm in one determined strategy if its reaction function coefficient is statistically significant at 90% or above confidence level in 5 or more regressions out of 7.

<sup>7</sup> This argument would not work in the other direction - there is no apparent reason to increase prices when the benchmark firm decreases.

<sup>8</sup> The Nelder-Mead method is a simplex method for finding a local minimum of a function of several variables. For two variables, a simplex is a triangle, and the method is a pattern search that compares function values at the three vertices of a triangle. The worst vertex, which is the largest, is rejected and replaced with a new vertex. A new triangle is formed and the search is continued. The process generates a sequence of triangles (which might have different shapes), for which the function values at the vertices get smaller and smaller. The size of the triangles is reduced and the coordinates of the minimum point are found.

The algorithm is stated using the term simplex (a generalized triangle in n dimensions) and will find the minimum of a function of n variables. It is effective and computationally compact. For a function of n variables, the algorithm maintains a set of n+1 points forming the vertices of a simplex or polytope in n-dimensional space. This simplex is successively updated at each iteration by discarding the vertex having the highest function value and replacing it with a new vertex having a lower function value. Such direct search methods have the advantage of requiring no derivative computations (indeed, the objective function need not even be smooth), but they tend to be efficient only in relatively low dimensions.

Firms start to maximize from period 3 to 9 and period 1 and give the initial conditions to our algorithm.