Testing GARCH and RV Exchange Rate Volatility Models using Hinich Tricorrelations

Sanja S. Dudukovic
Franklin University – Switzerland

Abstract
The aim of this paper is to enlighten a need to test the two most popular volatility models: the GARCH-ARMA model, based on a daily returns and the RV and the ARMA model, based on 30 min intraday high frequency (HF) data, in terms of non Gaussian Time Series Analysis. The ability of the models to perform a digital whitening and to produce independent innovations is tested on seven foreign exchange rates (FX) including Jpy/Eur, Usd/Eur, Cad/Usd, Chf/Eur, Chf/Usd, Usd/Gbp and Gbp/Eur, taken from Bloomberg. In the first step, stationary ARMA-GARCH models of different orders were built and the best model was chosen by using AIC and Box-Pierce test based on the innovations of daily squared returns. In the second step, realized daily volatilities (RV), defined as the sum of intraday squared 30 min returns, are used to estimate the RV-ARMA volatility model parameter and to calculate forecasting errors. In the third step, the higher order cumulants (HOC) are calculated for 20 lags for all currencies and used to perform the Hinich test. Finally, it was not shown that whitening of squared returns, neither by using GARCH-ARMA nor by using RV ARMA model, is efficient. The finding of serial dependence in innovations signifies the presence of structure in the data that cannot be modeled by ARCH or GARCH or RV volatility models that assume a pure noise input. A further improvement is suggested in the stage of parameter estimation by using Higher Order Cumulant function , prior to the model testing based on Hinich test.

Keywords: Volatility Forecasting, , Higher Order Cumulant Function , GARCH model, ARMA Model, Exchange rate Volatility , Model testing , Hinich test, Realized Volatility.

JEL Classification Numbers: G15, G17

Introduction
Amid the current financial and banking crisis, there is a long standing discussion about the origins of crisis in general and the currency crisis in particular. An important empirical condition which bears witnesses of the crisis is the fact that the real FX probability density functions (pdf) is leptokurtic with a fat tails. In addition, volatility clustering often characterize market returns, which means that periods of a high volatility are followed by periods of a high volatility and periods of a low volatility are followed by periods of a low volatility. This implies that the past volatility could as a predictor of the volatility in the next period. It was believed that both volatility clustering and fat tails could be explained by using the well-known Generalized Autoregressive Conditional Heteroskedasticity (GARCH) or Realized Volatility (RV) model.

Indeed, there are many variants of ARCH/GARCH models which are developed to improve the out-of-sample volatility forecasting performance. These models have many strong proponents who believe that GARCH models are currently the best obtainable forecast estimators. There are studies which confirm a very low coefficient of determination produced by GARCH models. For instance, Anderson and Bollerslev (2001), Poon & Granger (2003), Carrol & Kearney (2009) showed theoretically that R^2 for a GARCH(1,1) model tends to 1/□, where □ stands for the kurtosis of the distribution of stock returns. This means that the highest coefficient of determination for Gaussian returns achievable by GARCH models is bounded from above by 1/3.

Contrary to the standard setting for economic prediction evaluation, the volatility is not directly measurable, but rather intrinsically latent. Accordingly, any ex post evaluation of forecast accuracy must account for a fundamental data error problem. The availability of the high-frequency intraday data has led a number of recent studies to endorse the use of so-called realized volatilities (RV), constructed from the summation of finely sampled high-frequency squared returns, as a practical method for improving the ex post volatility measures. Assuming that the sampling frequency of the squared returns utilized in the realized volatility computations approaches zero, the realized volatility is then believed to estimate consistently the true (latent) integrated volatility.

Market microstructure noise is produced by a range of inbuilt frictions built into a trading system: bid–ask bounces, discreteness of price changes, differences in trade volume or information content of price changes, steady response of prices to a block trade, etc. In fact, market structure effect introduces a bias that grows as the sampling frequency increases. This motivated the idea of viewing the observed FX price as noisy measures of the latent true FX spot price. These tick-by-tick return processes obviously violate the theoretical semi-martingale restrictions implied by the no-arbitrage assumptions in continuous-time asset pricing models. Thus realized volatility, constructed directly from ultra high frequency returns, appears to be regularly corrupted with the measurement error (Bai, Russell, and Tiao (2000)).

Corsi (2009) introduced a simple Heterogeneous Autoregressive model of the Realized Volatility (HAR), which directly model and forecast the time series behavior of volatility realized over different time horizons. Corsi demonstrated that his model successfully reproduced some of the main empirical characteristics of the financial data – fat tails and long memory in the volatility – by using tick-by-tick logarithmic middle prices of USD/CHF FX rates in the period of 12 years (from December '89 to July 2001). Hansen &Huang (2011) introduced a Realized GARCH model with the aim to include the relation between the observed realized volatility and the latent volatility. They demonstrated a substantial improvement in the log-likelihood in the case of DJIA index.

Traditional methods of comparing volatility models insofar have been: Mean Forecast Error (MSE) produced by those models and its many variants; maximum likelihood value and AIC or BIC criteria. Most recently the other approach is taken . That is to say, given a set of characteristic features or exchange rate stylized facts such as volatility clustering, fat tail phenomena, leverage effect, or Taylor effect, one may ask the following question: have popular volatility models been parameterized in such a way that they can accommodate and explain the most common stylized facts visible in the data? Models for which the answer is positive may be viewed as suitable for practical use. For example, Teräsvirta (1996) investigated the ability of the GARCH model to reproduce series with high kurtosis and, at the same time, positive but low and slowly decreasing autocorrelations (AC) of squared observations. Bai, Russell & Tiao (2003) also compared GARCH and ARSV models in terms of kurtosis -autocorrelation relationship in squared returns.

Ultimately, non Gaussian Time Series Analysis (NGTSA) is gaining new importance in the context of volatility modeling and risk management. Ideally, in terms of NGTSA, a good volatility model should have a capacity to perform “digital whitening“ of stock market squared returns and therefore to produce independent and identically distributed innovations (iid), which are known as forecasting errors or simply as driving noise (Lim & Hinich (2006).

The aim of this paper is to test the ability of two best known volatility models, GARCH and RV, to produce non correlated and independent innovations. The organization of the paper is as follows. The GARCH and the RV models are defined in Section 2; The Box& Pierce test and the Hinich tricorrelation test are introduced in Section 3.The same section presents the introduction to higher order moments and cumulants. The data description and model building results are presented in Section 3. Section 4 presents comparative innovation analysis and model testing results. Section 5 contains conclusions and suggestions for further research.

Volatility Models

The fact that stock market returns are often characterized by volatility clustering – which means that periods of a high volatility are followed by periods of a high volatility and periods of a low volatility are followed by periods of a low volatility – implies that the past volatility could be used as a predictor of the volatility in the next period. Although the autocorrelation of the returns is insignificant at all frequencies, the autocorrelations of the squared absolute returns persist within a very long time interval demonstrating a long memory in volatility.

The GARCH Model

Let e_t denote a discrete time stationary stochastic process. The GARCH (p, q) process is given by the following set of equations:

$$r_t = \log(P_t) - \log(P_{t-1}) \tag{1}$$

$$r_t = x(k)g(k) + e_t \tag{2}$$

$$e_t = v_t\sqrt{h_t}$$

$$e_t/_{t-1} \approx N(0, h_t) \tag{3}$$

$$h_t = \alpha_0 + \sum_1^p \alpha_i e^2_{t-i} + \sum_1^q \beta_j h_{t-j} \quad , \tag{4}$$

where p_t represents stock prices; e_t represents random returns; $x(k)$ is a vector of explanatory variables; $g\ (k)$ is a vector of multiple regression parameters; h_t is the conditional volatility; α_i is autoregressive; and β_j is the moving average parameter as related to the squared stock market index residuals. An equivalent ARMA representation of the GARCH (p, q) model (Bollerslev, 1982, pp. 42-56) is given by:

$$e_t^2 = \alpha_0 + \sum_1^p(\alpha_i + \beta_i)e^2_{t-i} + \nu_t - \sum_1^q \beta_j \nu_{t-j} \quad , \qquad \nu t = e_t^2 - h_t \tag{5}$$

where h_t is known as GARCH variance.

In this context, the GARCH (p, q) volatility model is simply an Autoregressive Moving Average, ARMA (p,q) model in e_t^2 driven by iid noise $\nu_{t,}$ which is Gaussian random variable. It is worth stressing that the GARCH variance, h_t, in time series analysis, appears to be merely an estimate of the squared de-trended SM returns e_t^2.

The best known ARMA model building methodology is the Box-Jenkins (1976) iterative methodology, which includes three steps: model order determination, parameter estimation and model testing. This methodology assumes that each stationary time series can be treated as an output from the AR(p), MA(q) or ARMA (p,q) filter, which has as an input – uncorrelated and Gaussian innovations – known as "white noise" {□□}.

The ARMA model has the following form: A(Z) e_t^2= B(Z)* □$_t$, where **Z** is a backward shift operator: : $e_{t-1}^2=Z^{-1}\ e_t^2$: $e_{t-k}^2 =Z^{-k}\ e_t^2$, and where $A(Z)=1-\alpha_1 Z^{-1}-\alpha_2 Z^{-2} - \alpha_p Z^{-p}$ and $B(Z)=1-\beta_1 Z^{-1}-\beta_2 Z^{-2} - \beta_q Z^{-q}$ are characteristic functions of orders p and q respectively. The roots of the characteristic functions of the ARMA model must be within the unit cycle to guarantee stationarity and invertibility of the model.

Realized Volatility Models

Recently Corsi (2009) introduced an alternative approach to construct an observable proxy for the latent volatility by using intraday high frequency data. His work was inspired by Merton (1980) Merton inspired his work, who showed that the integrated volatility of a Brownian motion can be approximated to an arbitrary precision using the sum of intraday squared returns.

$$IV_t=\int_{t-1}^{t} \sigma^2(s)ds \tag{6}$$

So, in this integrated framework, the Integrated Variance (IV) is considered to be the population measure of actual return variance. Namely, it was proved that the sum of intraday squared returns converges (as the maximal length of returns go to zero) to the integrated volatility of the returns, making it possible to construct an error free estimate of the actual volatility. This nonparametric volatility estimator is known as realized volatility (RV).

$$RV_t=\sum_{1}^{\infty}\ r^2_{t,i} \tag{7}$$

Taken accurately, this theory suggests that one should sample prices as often as possible. This would lead to estimate IV_t by RV_t from tick-by-tick data. However, as was noted in Merton (1980) “in practice, the choice of an even-shorter observation interval introduces another type of error which will swamp the benefit long before the continuous limit is reached”. The modern terminology for this phenomenon is known as market microstructure effects. These effects cause the observed market price to diverge from the efficient price. All told, market structure effect introduces a bias that grows as the sampling frequency increases. This motivated the idea of viewing the observed prices, p_t, as noisy measures of the latent true price.

Indeed, in practice, empirical data at very small time (RV) make a strongly biased estimator in case of small SM return interval. Therefore, a trade-off between measurement error and bias must be found. On one hand, statistical theory would compel a very high number of return observations to reduce the stochastic error of the measurement. On the other hand, market microstructure comes into play, introducing a bias that grows as the sampling frequency increases. Given such a trade-off between, a simple way out is to choose, for each financial variable, the shortest return interval at which the resulting volatility is still not significantly affected by the bias. This approach has been pursued by Andersen et al. (2001), who agree on a return interval of 30 minutes for the most highly liquid exchange rates, leading to only 48 observations per day.

Literally, it is believed that the realised volatility, defined as the sum of intraday, 30 min squared returns, provides a more accurate estimate of the latent volatility than the estimate based on daily squared returns.

$$RV_t=\sum_1^{48} r^2_{t,i} \quad (8)$$

The theoretical and empirical properties of realized volatility are derived in (Andersen, Bollerslev, Diebold and Labys, 2001) for foreign exchange. They found that realised volatility distribution is nearly Gaussian. In this article the ARMA model applied to RV is tested :

$$RV_t = \alpha_0 + \sum_1^p \alpha_i RV_{t-i} + u_t - \sum_1^q \beta_j u_{t-j} \quad (9)$$

Estimated realized volatility is then calculated by using the formula: ERV $=RV_t-u_t$

Model Testing Methods

There are two tests which can be applied to test the null hypothesis that the ARMA model innovation time series represent a white noise. The first is the well known Box& Pierce test which can be applied if innovation series – driving noise is independent and identically distributed Gaussian process. In the case of non Gaussian probability density function, the Box-Pierce test would not show model inadequacy since it is based only on second order statistics, which is no longer sufficient for parameter estimation.

All stationary time series are time reversible (TR). The contrary is not true. Visually, TIR demonstrate a tendency of a variable to rise rapidly to local maxima and then to decay slowly. This time reversibility amounts to temporal symmetry in the probabilistic structure of the process. TR cannot be evaluated by using the second order cumulants – autocovarianvce function. Therefore the Hinich test based on higher order cumulant function is more appropriate in non Gaussian case.

Box-Pierce Q Test

As for diagnostic checking, if obtained model is appropriate and the parameter estimates are consistent and efficient for the particular time series, then the model innovations t would be uncorrelated random deviates, and their first L sample autocorrelations:

$$AC(k) = \sum \nu(t)\nu(t-k) / \sum \nu \quad (10)$$

would have a multivariate normal distribution Box-Pierce (1970). They also showed that the AC(k), k=0,1,2…L, are uncorrelated with variances which could be approximated by :
Var(AC(k)) = n-k/n(n+2) ≈1/n, from which it follows specifically that $n(n+2)\sum(n-k)^{-1}AC(k)^2$ would, for large n, be distributed as $\chi 2$ with L degrees of freedom; or as a further approximation:

$$n\sum(AC(k)^2 \approx \chi_L^2 \quad (11)$$

When applied to the ARMA parameter estimation, degree of freedom must be changed to

L-p-q, where p and q are the orders of the autoregressive and moving average operators.

The Hinich Test

As proved empirically, in the case of exchange rate and stock market returns, driving noise is not independent and (most usually) it is non Gaussian either. Subsequently, the second order moment and correlation function do not represent "sufficient statistics", neither for the ARMA parameter estimation, nor for the model testing. In fact, it is well known that for a non-Gaussian process, the higher order moments exist and are different from zero.

Cumulants

Eversince it was realized that the normal distribution was unsatisfactory for describing economic and demographic data, there were attempts to use of a special type of distribution to represent a new system of skewed distributions f(z) by using the quantities of the distribution expressed as $□_r = □_{r+2}/□_2^{(r+2)/2}$ .Terms $□_r$ are called distribution invariants-cumulants , while $□_r$ are independent of both location and scale and therefore are called a distribution shape coefficients (Hald 1981, p. 7). So far, at least three forms of a general probability density distribution with a priory unknown shapes were proposed : Chebishev, Gram Charlie and Edgeworth. Insofar, the best properties in terms of integralability and convergence are found in Edgeworth distribution approximation. Its form allows any standard probability distribution f (z) to be expressed in terms of Gaussian distribution □ (z):

$$f(z) = \phi(z) + -\lambda_3\phi^3(\zeta)/3! + \lambda_4\phi^4(z)/4! + \lambda_3{}^2\phi^6(z)/6!...., \quad \text{where } z = (x-\kappa 1)/\sqrt{\kappa 2}.$$

In the area of digital signal processing, Giannakis (1990) was the first to show that the third and the sample fourth order cummulant functions , $C^3{}_r(□1,□2$ and $C^4(□1,□2,□3)$,can be estimated from the observed time series $\{r_t\}$:

$$C^3{}_r(\tau_1,\tau_2) = (\sum(r(t)r(t+\tau_1)r(t+\tau_2))/n, \qquad \tau_2=1,2..L\ ,\ \tau_1=1,2...L \qquad (12)$$

$$C^4{}_r(\tau_1,\tau_2,\tau_3) = (\sum(r(t)r(t+\tau_1)r(t+\tau_2)\ r(t+\tau_3))/n \quad \tau_2=1,2..3\ ;\ \tau_2=1,2..L\ ,\ \tau_1=1,2...L \qquad (13)$$

where n is a number of observations and the second-order cumulant $C^2{}_r(□)$ is just the autocorrelation function of the time series of returns $r_{t,}$ t=1,2.3…n.

2.2.2. *The Hinich test*

Hinich (1996) developed a test statistics aimed to check serial dependence in the innovation data by using auto correlation, bicorrelations and tricorrelations .The null hypothesis is that the ARMA model innovations are realizations of a pure white noise process. Therefore, under the null hypothesis, all $C^2(r)] = 0$, for all $r \neq 0$, the bicorrelations C^3 (r, s) =E[□(t) □(t- r)□(t- s)] for all r and s , except where r =s=0 , and the tricorrelations C^4 (r, s, v) =E[□(t)□(t -r)□(t- s)□(t -v)] =0 for all r ,s and v , except where r = s = v = 0

The H2 statistics, known as Q statistics, originally developed by Box-Pierce (10), is used to test linear serial dependence. H3 and H4 are designed to test for the existence of a higher order serial dependence (Wild, Foster and Hinich, 2010, pg 9):

H3=(n-s) $\sum_{s=2}^{L}\sum_{r=1}^{s-1}\{C^3\ (r,s)\}^2 \approx \chi 2$ with L(L-1)/2 d.f. , where L is number of lags. (14)

H4 =(n-v)$^{3/2}$ $\sum_{v=2}^{L}\sum_{s=2}^{v-1}\{\sum_{r=1}^{s-1}\{C^3(r,s,v)\}^3 \approx \chi 2$ with L(L-1)(L-2)/3.f. (15)

The number of lags L is defied as L=n^b , with 0<b<.5 , for the H2 and H3and <0<b<.33 for the test based on the forth order cumulants. According to Wild , Foster and Hinich (2010) , "if the null hypothesis of pure noise is rejected by the H2, H3 or H4 tests , this then signifies the presence of structure in the data that cannot be modeled by ARCH or GARCH or stochastic volatility models that assume a pure noise input.

Empirical Analysis

GARCH-ARMA results

The ARMA-GARCH empirical analysis is based on daily quotations of closing daily exchange rates for the period from Sep 21, 2012 to March 20, 2013, taken from Bloomberg. The common sample of exchange rate description is presented in Table 1.

The reported statistics confirm the skewed distributions across all currencies. In addition, the sample kurtosis for each currency is well above the normal value of 3. Jarque-Bera values show that all FX return distributions are leptokurtic and depart significantly from the Gaussian distribution.

The ARMA-GARCH parameter estimates based on OLS method are given in Table 2. The table presents only the best stationary model for each currency and is chosen when achieving the minimum Akaike Information Criterion (AIC).

The Box &Pierce (1980) test of the null hypothesis that the first K autocorrelations of covariance stationary innovations are zero, in the presence of statistical dependence, was performed . The results are in Table 2. The presented results show the best ARMA model for each currency in terms of AIC criterion. All selected models are stationary. Stationarity is achieved by taking the first or the second difference of FX returns. ARCH-ARMA parameter estimates show unexpectedly high coefficient of determination. Q statistics shows that residuals are non-correlated. Estimated GARCH volatilities calculated by using (5) are presented in fig 1.

The statistical properties of GARCH innovations are given in Table 3. As it can be seen from the table, kurtosis is extremely high for all currencies, which suggests a strong departure from the model assumption, which stated that GARCH residuals were supposed to be normally distributed.

RV-ARMA Empirical Results

High frequency squared returns were used to create daily realized volatility for all currencies (8). RV data are presented in Figure 2.Their statistical properties are given in Table 4.The kurtosis of the returns is much higher than that of a normal distribution at intraday frequency and tends to decrease as the return length increases. Thus return probability density functions (pdfs) are leptokurtic with a fat tails. Those stylized facts are not seen only in autocorrelation function, kurtosis and skewness of squared returns, but also in the third and the fourth order cumulant functions. Table 4 clearly shows departure from the Gaussian distribution. These realised volatilities are then used to make an RV-ARMA (p,q) model. The model parameters based on E-views software are presented in Table 5.

As it can be seen from Table 5, Box-Pierce test statistics Q, applied to model residuals, shows that for two currencies hypothesis of non correlated innovations cannot be rejected . This finding suggests a question of the validity of the assumption that RV residuals are produced by a non correlated noise.

Comparative Analysis

Proceeding with the data description, statistical properties of the GARCH-ARMA residuals and RV-ARMA innovations are presented in Table 6.1 and Table 6.2 respectively. From statistical description, it is obvious that neither of the volatility models has captured high kurtosis of squared daily returns or realized daily volatility in the case of seven currencies being tested. This contradicts the finding by (Andersen, Bollerslev, Diebold and Labys, 2000) that residuals are nearly Gaussian.

The forth order cumulants of squared returns and corresponding ARMA GARCH innovations are calculated by using (10) and (11) and marked by "RESR2GDPEUR" for example . The fourth order cumulants of corresponding realized volatility, ARMA-RV innovations, are also calculated for each currency by using (9). The cumulants for seven currencies are presented and are presented in Figures 3.1, 3.2 and 3.4.These figures demonstrate that residual cumulants are different from zero, i.e., that both ARMA-GARCH and ARMA-RV volatility models produce non Gaussian innovations without being able to capture the stylized facts from FX returns.

Ultimately, to substantiate analysis, the null hypothesis that innovation- driving noise is iid process is tested using Hinch test. The results are presented in Table 7.The results show that in the case of four RV-ARMA models, the null hypothesis which states that innovations are "white", cannot be rejected . But in five cases of GARCH-ARMA innovations, Hinich test values were higher than χ^2 critical, confirming that white innovations are not produced .

Conclusion

This paper aimed to compare ARMA-GARCH and ARMA-RV non Gaussian volatility models in terms of the statistical properties of their innovations on which both models are footing. Therefore, its objective was to test if the model innovations are white in terms of higher order cumulants, as in the case when the model completely extracts information necessary to forecast volatility.

Empirical analysis is based on seven foreign exchange rates (FX), including Jpy/Eur, Usd/Eur, Cad/Usd, Chf/Eur, Chf/Usd, Usd/Gbp and Gbp/Eur, taken from Bloomberg. The residual testing was performed by using Hinich triple correlation test, which is based on the third and on the fourth order cumulant functions. The concept of cumulants is also introduced . The results demonstrated that neither ARMA-GARCH nor RV-GARH, if estimated by using the second order statistics, produce "white" innovations.

It was confirmed that, if there are both third- and fourth-order nonlinear serial dependence in the data, time series models that make use of a linear structure or presume a pure white noise input (such as the geometric Brownian motion (GBM) stochastic diffusion model) are problematic. In particular, the dependence structure violates both the normality and Markovian assumptions underpinning conventional GBM models.

This paper's finding of serial dependence in GARCH and RV innovations has important implications for the use of GBM and jump diffusion models that currently emphasize accepted risk management strategies based on the Black–Scholes option pricing model, which are employed in financial and investment management.

Therefore, the question of non Gaussian parameter estimation in volatility forecasting remains an everlasting problem which definitely needs to be addressed in terms of a HOC parameter estimation methodology.

References

Andersen, T. G., Bollerslev, T., Diebold, F. X. and Labys, P. 2000. Exchange Rate Returns Standardized by realized volatility are nearly Gaussian. Multinational Financial Journal. 4,159-179.

Anderson T.W. and Darling D.A. 1952. Asymptotic Theory of Certain Goodness of Fit Criteria Based on Stochastic Processes. Annals of Mathematical Statistics. 23(2), 169-313

Bai X., Russell J.R.,. Tiao C.G.2000. Beyond Merton's Utopia (I): effects of non-normality and dependence on the precision of variance estimates using high-frequency financial data. .Retrieved from: http://faculty.chicagobooth.edu/jeffrey.russell/research/merton1.pdf

Bai, N., Russell, J. R., & Tiao, G. C. 2000. Kurtosis of GARCH and stochastic volatility models with non-normality. Journal of Econometrics, 114, 349–360.

Bollerslev, T. (1982). Generalized Autoregressive Conditional Heteroskedasticity, in *ARCH Selected Readings*, ed. by Engle, R. Oxford: Oxford UP, 42-60.

Box, G. E. P. and Pierce, D. A.1970. Distribution of Residual Autocorrelations in Autoregressive-Integrated Moving Average Time Series Models. Journal of the American Statistical Association, 65: 1509–1526. JSTOR 2284333

Corsi, F.2009 .A Simple Approximate Long Memory Model of Realized Volatility. Journal of Financial Econometrics 7: 174-196

Carrol R. and Kearvey C.2009. GARCH Modeling of stock market volatility in Gregoriou G. (ed), Stock Market Volatility, CRC Finance Series, CRC Press, 71-90.

Hansen, P. R. & Z Huang. 2011. Realized GARCH: A Joint Model of Returns and Realized Measures of Volatility. Journal Of Applied Econometrics .Published online : http://public.econ.duke.edu/~get/browse/courses/201/spr12/DOWNLOADS/WorkingPapers_Now_Published/phs_realized_garch_10.pdf

Hinich, M.J. (1982), 'Testing for Gaussianity and Linearity of a Stationary Time Series', Journal of Time Series Analysis, 3: 169–76.

Hinich, M.J. (1996) Testing for dependence in the input to a linear time series model. Journal of non parametric Statistics 6, 205–221

Lim K.P., Hinich M. and Liew V. K. (2006): Statistical Inadequacy of GARCH Models for Asian Stock Markets: Evidence and Implications .Journal of Emerging market Finance, 4(3) Retrieved from http://hinich.webhost.utenas.edu/files/economics/asian-garch.pdf

Lobito I., Nankervis J. & Savin. N. Testing for Autocorrelation Using a Modified Box-Pierce Q Test International Economic Review.2001.42 (1), 187-205

Merton, R.C. (1980), On Estimating the Expected Return on the Market: An Exploratory Investigation, Journal of Financial Economics, 8, 323-361.

Poon H. and Granger C., (2003), Forecasting Volatility in Financial market: A review, Journal of Economic Literature, Vol. XLI, 478-539

Teräsvirta T Zhao Z. (2011).Stylized Facts of Return Series, Robust Estimates, and Three Popular Models of Volatility. *Applied Financial Economics* Volume 21, Issue 1-2, pp.67-94.

Wild P., Fostera1 J, Hinich M. (2010). Identifying Nonlinear Serial Dependence in Volatile, High-Frequency Time Series And its Implications For Volatility Modeling. Macroeconomic Dynamics. 14 (5), 88-110

Appendix A1

Table 1. Descriptive statistics of the squared FX daily returns

	R2JPYEUR	R2USDEUR	R2CADUSD	R2CHFEUR	R2CHFUSD	R2USDGBP
Mean	0.15	0.04	0.02	0.01	0.03	0.03
Median	0.05	0.02	0.01	0.00	0.01	0.01
Maximum	1.36	0.47	0.15	0.28	0.28	0.29
Minimum	0.00	0.00	0.00	0.00	0.00	0.00
Std. Dev.	0.25	0.06	0.03	0.03	0.05	0.05
Skewness	3.12	4.06	2.45	5.25	2.62	2.81
Kurtosis	13.50	26.38	8.87	35.47	10.11	12.57
Jarque-Bera	677.65	2782.05	265.30	5288.11	353.76	559.04

Table2. ARMA-GARCH parameter estimates.

Currency	C	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	MA(1)	MA(2)	MA(3)	MA(4)	MA(5)	R2	AIC	Q
JPYEUR	0.159	0.240	-0.163	0.471	-0.231	0.544	-0.226	0.232	-0.693	0.450	-0.407	0.222	-0.115	36.166
st.error	0.059	0.306	0.259	0.148	0.218	0.243	0.337	0.281	0.123	0.292	0.329			
USDEUR*		-1.877	-1.844	-0.846	-0.037		0.968	0.025	-0.938	-0.951		0.575	-2.781	25.418
		0.094	0.180	0.180	0.093		0.023	0.018	0.021	0.024				
CADUSD	0.010	0.498	-0.879	0.880	-0.449	0.924	-0.476	0.965	-0.952	0.470	-0.970	0.124	-3.327	31.493
	0.022	0.027	0.031	0.023	0.028	0.028	0.022	0.017	0.014	0.021	0.016			
CHFEUR		0.281	0.370	-0.317	-0.110	0.699	-0.125	-0.416	0.603	0.174	-0.795	0.222	-4.803	26.896
		0.077	0.092	0.080	0.078	0.065	0.075	0.081	0.045	0.070	0.067			
CHFUSD*		-1.130	-0.875	-1.051	-0.974	-0.124	0.161	-0.288	0.176	-0.037	-0.871	0.528	-2.397	32.374
		0.101	0.130	0.114	0.087	0.067	0.078	0.075	0.094	0.072	0.072			
USDGBP*		0.831	-0.938	0.193	0.043		-1.959	2.017	-1.406	0.407		0.572	-3.271	25.653
		0.161	0.138	0.143	0.083		0.171	0.265	0.261	0.143				

*denotes a first difference ARMA model

Table 3. ARMA-GARCH innovation statistical description

	RESR2JPYE	RESR2USD	RESR2CAD	RESR2CHF	RESR2CHF	RESR2USD	RESR2GBPEUR
Mean	0.0057	0.0023	-0.0027	0.0030	-0.0032	0.0051	0.0093
Median	-0.0534	-0.0080	-0.0122	-0.0005	-0.0180	-0.0071	-0.0050
Maximum	0.9920	0.3978	0.1251	0.2714	0.2550	0.2368	0.4167
Minimum	-0.4392	-0.0892	-0.0363	-0.0418	-0.0730	-0.0760	-0.2269
Std. Dev.	0.2149	0.0557	0.0311	0.0313	0.0547	0.0448	0.0785
Skewness	2.0918	3.8307	2.0813	6.1100	2.6506	2.2811	2.1602
Kurtosis	9.8540	25.6201	7.6652	51.4944	11.0768	10.4969	12.2986
Jarque-Bera	292.8428	2590.4050	177.5413	11358.8500	423.9068	349.7843	477.4624

Table 4. Statistical description of daily realized volatilities

	RVJPYEUR	RVUSDEUR	RVCADUSD	RVCHFEUR	RVCHFUSD	RVUSDGBP	RVGBPEUR
Mean	0.46	0.10	1.10	0.85	0.17	0.24	0.15
Median	0.28	0.06	1.05	0.84	0.11	0.14	0.09
Maximum	3.37	0.75	2.19	1.07	0.98	1.64	1.07
Minimum	0.06	0.01	0.02	0.12	0.01	0.02	0.01
Std. Dev.	0.47	0.11	0.19	0.08	0.19	0.28	0.16
Skewness	2.90	3.06	0.78	-6.14	2.26	2.44	2.86
Kurtosis	15.35	15.09	21.32	62.19	8.40	10.36	13.70
Jarque-Bera	907.8	896.1	1648.0	17815.6	241.5	380.2	717.2

Table 5: RV-ARMA parameters

	C	AR(1)	AR(2)	AR(3	AR(4)	AR(5)	AR(6)	AR(7)	MA(1)	MA(2)	MA(3)	MA(4)	MA(5)	R^2	AIC:	Q:
JPYUR**		-1.53	-1.75	-1.70	-1.39	-1.00	-0.60	-0.25						0.74	1.90	24.03
		0.09	0.16	0.21	0.23	0.21	0.16	0.09								
USDEUR	0.10	0.40	0.26	-0.41	-0.34				-0.59	-0.24	0.62	0.51		0.29	-1.88	20.01
	0.01	1.06	1.11	0.36	0.46				1.04	1.31	0.53	0.62				
CADUSD	0.88	-0.05	0.67	0.43	-0.30	0.23			0.45	-0.59	-0.77	0.03	-0.10	0.35	-0.41	217.36
	0.16	0.64	0.33	0.47	0.54	0.16			0.64	0.58	0.33	0.71	0.24			
CHFEUR	0.85	-0.35	0.91	-0.06	-0.88	-0.12			0.44	-0.91	0.00	1.03	0.22	0.08	-2.56	23.01
	0.01	0.25	0.05	0.25	0.09	0.22			0.26	0.04	0.26	0.08	0.25			
CHFUSD*		-0.75	-0.98	-0.69	-0.95	-0.09			-0.29	0.27	-0.23	0.34	-0.95	0.60	-2.37	108.99
		0.07	0.05	0.07	0.05	0.07			0.02	0.02	0.02	0.02	0.02			
USDGBP*		-0.23	-1.36	-0.26	-0.91	-0.12			-0.86	1.30	-1.22	0.91	-0.92	0.62	-3.24	25.41
		0.07	0.02	0.10	0.02	0.07			0.03	0.01	0.04	0.02	0.03			
GBPEUR		-0.28	-0.29	-0.50	-0.63	0.16			-0.49	-0.10	0.22	0.28	-0.87	0.46	-0.81	
		0.21	0.18	0.16	0.19	0.12			0.19	0.25	0.23	0.22	0.17			

*denotes an ARIMA model

** denotes a second difference model--i.e. d(rvjapeur,2)

Table 6.1. ARMA-GARCH innovation statistical description

	RESR2JPYEU	RESR2USDEU	RESR2CADU	RESR2CHFEU	RESR2CHFUS	RESR2USDG	RESR2GBPEUR
Mean	0.01	0.00	0.00	0.00	0.00	0.01	0.01
Median	-0.05	-0.01	-0.01	0.00	-0.02	-0.01	0.00
Maximum	0.99	0.40	0.13	0.27	0.26	0.24	0.42
Minimum	-0.44	-0.09	-0.04	-0.04	-0.07	-0.08	-0.23
Std. Dev.	0.21	0.06	0.03	0.03	0.05	0.04	0.08
Skewness	2.09	3.83	2.08	6.11	2.65	2.28	2.16
Kurtosis	9.85	25.62	7.67	51.49	11.08	10.50	12.30
Jarque-Bera	292.84	2590.41	177.54	11358.85	423.91	349.78	477.46

Table 6.2 RV-ARMA innovations description

	RESRVJPYEUR	RESRVUSDEUR	RESRVCADUSD	RESRVCHFEUR	RESRVCHFUSD	RESRVUSDGB	RESRVGBPEUR
Mean	0.00	0.01	-0.01	-0.02	-0.02	0.07	0.01
Median	-0.03	0.00	0.00	-0.05	-0.06	-0.07	-0.03
Maximum	0.66	0.76	0.21	0.45	1.14	3.15	0.79
Minimum	-0.11	-1.01	-0.71	-0.33	-0.47	-0.64	-0.27
Std. Dev.	0.11	0.16	0.08	0.14	0.24	0.52	0.15
Skewness	3.26	-1.14	-6.05	1.21	1.63	2.48	2.05
Kurtosis	17.75	23.03	58.53	5.03	8.37	13.61	9.74
Jarque-Bera	1180.31	1845.56	14669.51	45.08	179.59	622.56	282.62

Table 7. Hinich triple correlation test results for ARMA-RV and ARMA-GARCH innovations

	Innovations	
HOC-TEST	ARMA-GARCH	ARMA-RV
L=10 , Hcrit=73.11		
HINICHRESJPYEUR	121.407	41.319
HINICHRESCHFUSD	84.603	109.996
HINICHRESCHFEUR	132.371	337.439
HINICHRESCADUSD	93.203	11.302
HINICHRESUSDGBP	156.300	493.819
HINICHRESGBPEUR	63.901	42.374
HINICHRESUSDEUR	133.683	6.575

Appendix A2

Figure 1. ARMA-GARCH estimated daily variances

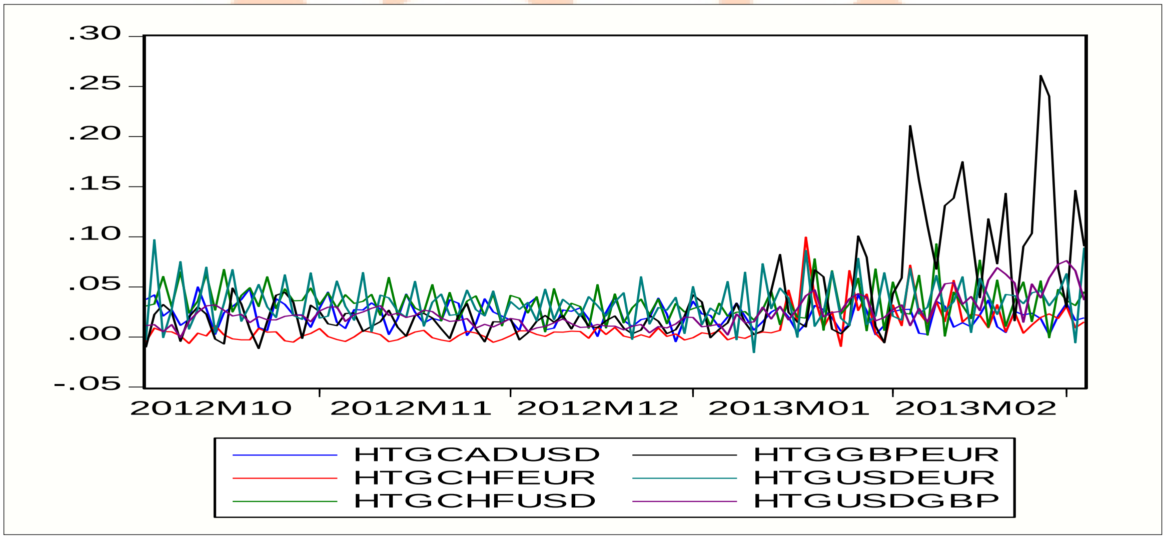

Figure 2. Daily realized volatility – all currencies

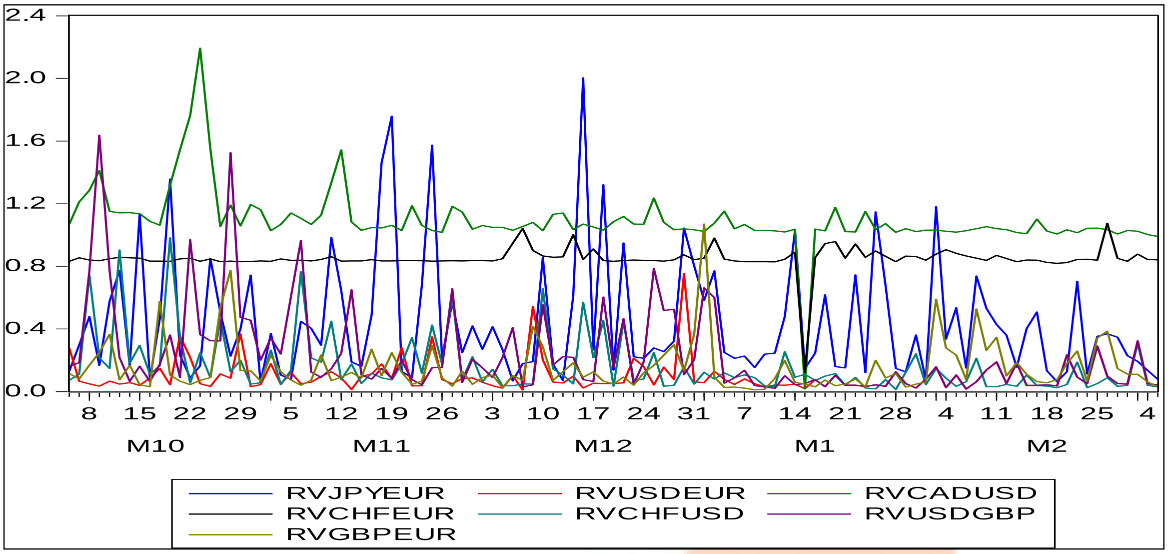

Figure 3.1 Fourth Order Cumulants: JPYEUR and CHFUSD GARCH and RV Innovations

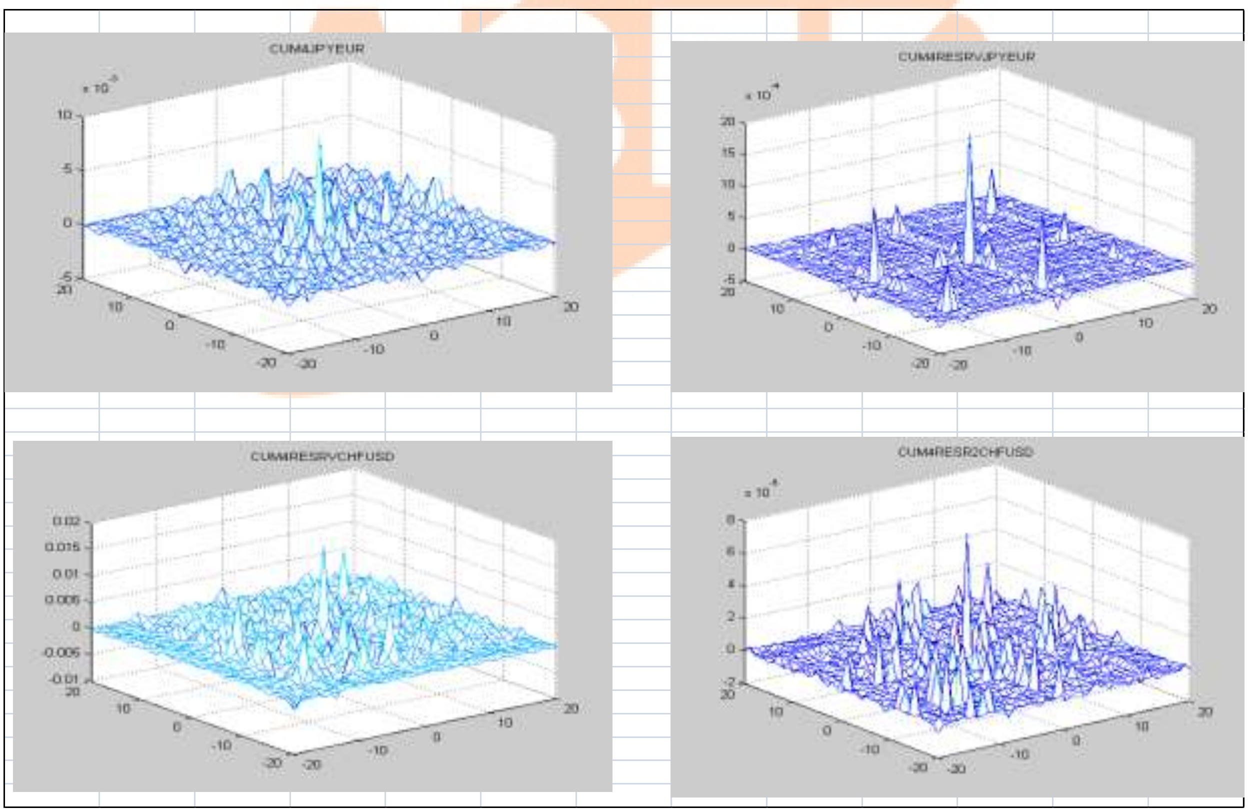

Figure 3.2 Fourth Order Cumulants: CHFEUR and CADUSD GARCH and RV Innovations

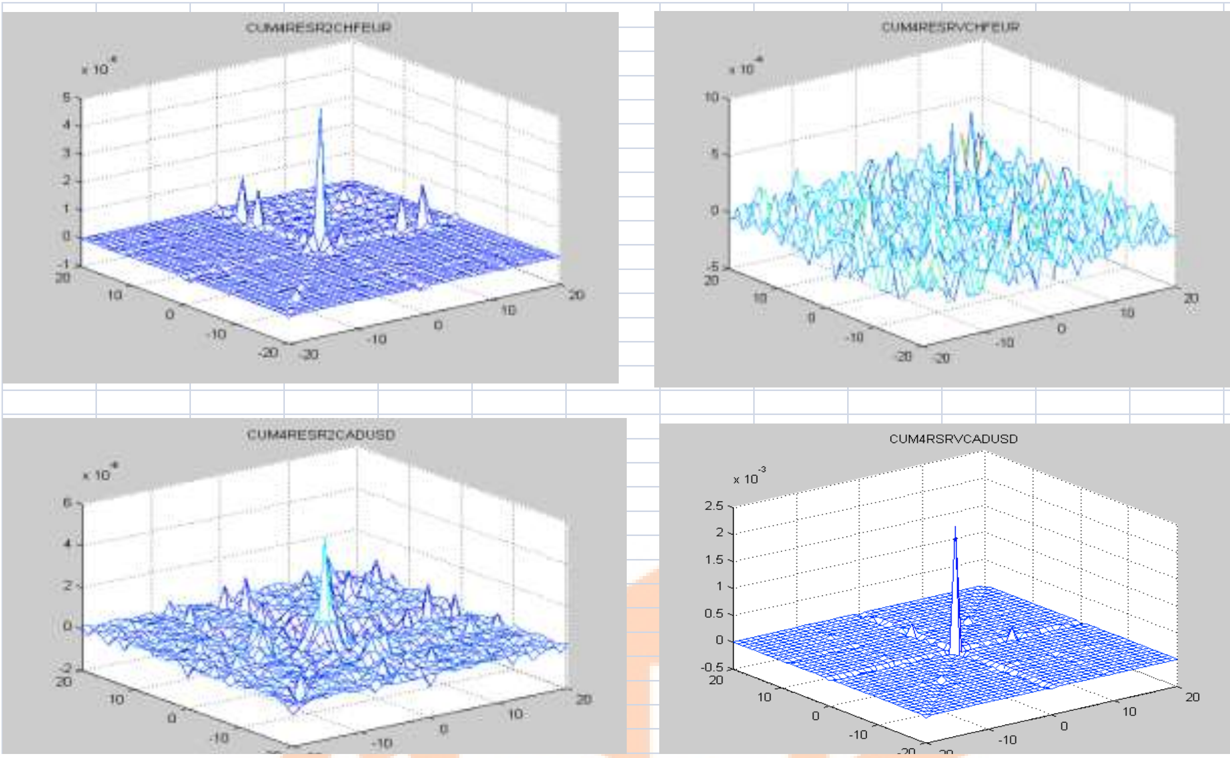

Figure 3.3 Fourth Order Cumulants –USDEUR and USDGBP GARCH and RV Innovations

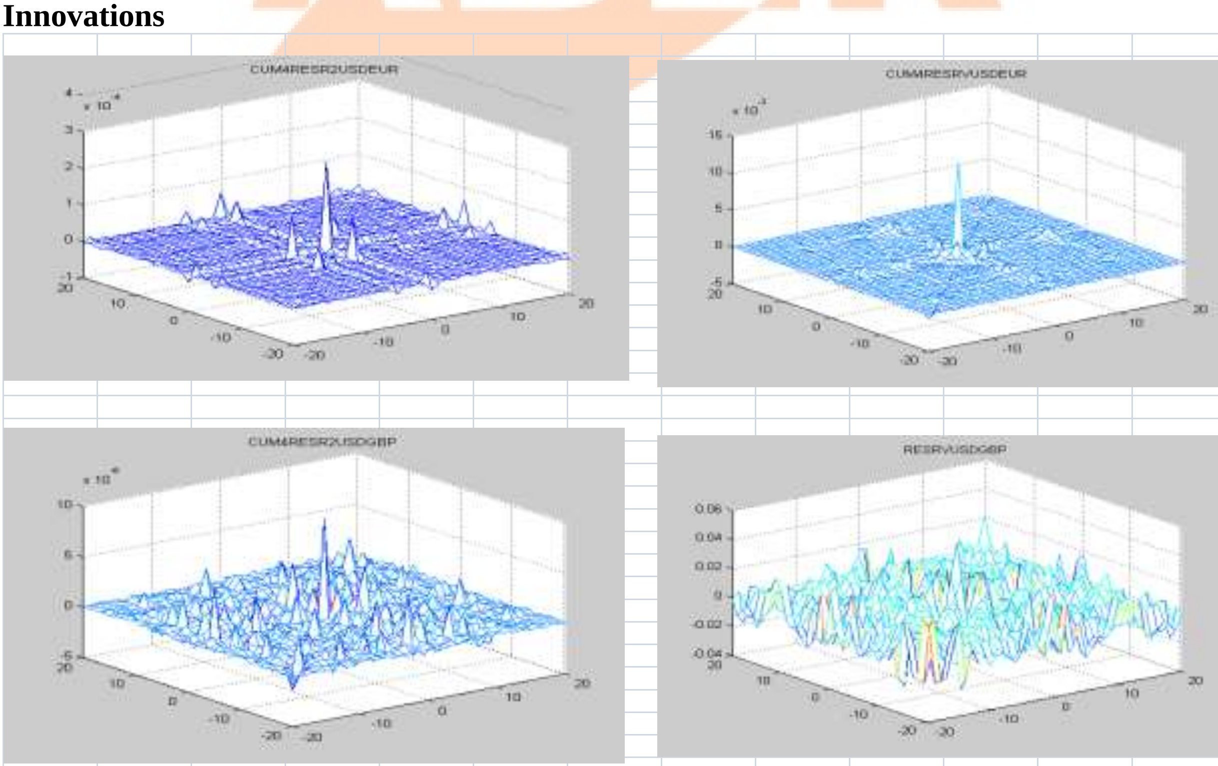

Figure 3.4 Fourth Order Cumulants – GBPEUR and GBPEUR GARCH and RV Innovations